A numerical study of convective heat transfer from an array of parallel blunt plates

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Two-dimensional (2-D) laminar flow and heat transfer through an array of parallel flat plates with finite thickness was studied numerically. A computer program based on the bounded skew hybrid differencing (BSHD) scheme was used and validated to determine the recirculation length, friction factor, and Nusselt number distribution along the plates. Computations were performed for a wide range of Reynolds number ($100 < Re_D < 400$), Prandtl number ($0.7 < Pr < 10.0$), and blockage ratios ($0.083 < Br < 0.40$). Results were compared for flow through an array of thin plates. Significant variations in friction factor and Nusselt number were noticed when the plate thicknesses were increased. For practical engineering applications, two correlations were developed: (1) the first correlation predicts the recirculation length in terms of the Reynolds number and blockage ratio; and (2) the second correlation determines the maximum Nusselt number close to the reattachment point in terms of the Reynolds number, Prandtl number, and blockage ratio.

Keywords: forced convection; laminar flow; blunt plates

Introduction

Laminar flow and heat transfer through an array of parallel blunt plates are of interest in a variety of applications, including surfaces with fins in heat exchangers, ribbed surfaces, and microchannels consisting of an interrupted array of stacked fins. As the flow moves from left to right (Figure 1), the boundary layer at the leading edge separates from the wall, forming a recirculation bubble, within which there is an adverse pressure gradient. Subsequently, the boundary layer reattaches to the wall and approaches the developing state.

Experimental studies of McCormick *et al.* (1984), revealed that, in general, details of heat transfer and pressure variations in separated flows depend upon the prior history of the upstream boundary layer. In such cases, potential flow influences are much more important and boundary-layer effects are typically negligible; that is, the separation occurs because of leading-edge bluntness, rather than because a boundary layer lacking in momentum encounters a sufficiently large adverse pressure gradient. The mixing in the region between the separation point and the reattachment point plays an important role in the heat and mass transfer processes and have been used positively in the design of heat exchangers.

For an array of parallel blunt flat plates, the essential parameters influencing the flow separation and reattachment are flow Reynolds number Re_D, and the blockage ratio Br. Small dimensions of these plates and small flow rates at which they operate in some cases result in a low-to-moderate Reynolds number corresponding to laminar flow.

An adequate theoretical description of heat and mass transfer at solid boundaries in the entrance region of parallel blunt plates with flow separation and reattachment is hampered by lack of a complete theoretical solution of fluid field and thermal structure. Land and Loehrke (1980) experimentally investigated the flow over an isolated blunt flat plate at low-Reynolds numbers, aligned parallel to the free stream. They visualized the flow streams using dye traces and observed a leading-edge separation bubble form at a Reynolds number of 80, based on the plate thickness. The bubble reached a maximum streamwise steady recirculation length of 6.5 times the plate thickness at $Re_D = 325$. At a higher Reynolds number, the separation shear layer became unsteady and turbulent (Tafti and Vanka 1991).

Numerical results for laminar flow over an isolated plate with a blunt leading edge was first reported by Djilali (1987). A finite-volume method was used in conjunction with two discretization schemes: the hybrid-upwind differencing (HD), and the bounded-skew-hybrid differencing (BSHD). It was observed that the reattachment length predicted with the HD scheme was up to 35% shorter than that observed experimentally. The BSHD predictions, on the other hand, were in excellent agreement with the experimental measurements of Land and Liehrke (1980). Later Coney *et al.* (1988a, b) published a series of papers on flow separation and reattachment. They reported experimental studies into separated flow, measuring the flow pattern and boundary-layer characteristics of downstream flow reattachment. Also, leading-edge separation for laminar incompressible flow is studied numerically more recently in terms of vorticity and stream function using the finite volume method by Kazeminejad *et al.*

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(1996). They developed a linear correlation for reattachment length in terms of Reynolds number based on the plate thickness. Their results for reattachment length agreed well with those measured by Land and Loehrke.

For all cases studied, the region of interest has been an isolated thick plate, placed in a large duct, while in many applications, the surfaces form stacks of plates aligned parallel to the free stream. Under this condition, fluid flow and heat transfer depend on the flow Reynolds number, plates spacing, and plate thickness. For such configuration, Djilali (1994) numerically studied fluid flow and heat transfer over the plates subjected to a constant surface heat flux. The separation bubble length and variation of the Nusselt number over the separation region for various Reynolds numbers and blockage ratios are presented graphically. He also found that the convection heat transfer increases substantially in the reattachment region, and a local maximum Nusselt number occurs slightly downstream of the reattachment point.

As far as the authors are aware, detailed theoretical work on flow through an array of parallel blunt flat plates at low-Reynolds number seems to be limited, and the enhanced mechanism attributable to recirculation in the laminar separated flow is still not fully formulated for practical applications. In these applications, the velocity and thermal boundary layer develop simultaneously, and the leading edge in entrance length depends on the recirculation length as well as the Graetz number. Furthermore, there is no direct comparison of the fluid flow and heat transfer for blunt plates with those of thin plates. Thin parallel plates have been studied extensively in the literature, while in most circumstances fluid enters from a reservoir or from an ambient through plates with finite thicknesses. A detail survey of laminar flow heat transfer in ducts with thin walls is reported by Shah and London (1978).

In this paper, the two-dimensional (2-D), steady, laminar, incompressible Navier–Stokes equations as well as energy the equation for separated and reattached flow through an array of bluff rectangular plates are solved numerically in terms of the primary variables using the bounded skew hybrid difference scheme (BSHD) (Raithby 1976; Lai *and* Gosman 1982). Velocity vector fields as well as temperature distributions are obtained for a range of low-Reynolds numbers $100 < \mathrm{Re}_D < 400$, and Prandtl numbers $070 < \mathrm{Pr} < 10.0$. The model development, solution technique, validation of the solution procedure, and selected results are discussed in the following sections.

Model development

The geometry and the coordinate system for the flow under consideration is illustrated in Figure 1. It is assumed that flow is uniform and parallel with the plates. Thermophysical properties of the fluid are assumed to remain constant, and the plates are relatively thick compared to the channel width. With these simplifying assumptions, the computational domain can be isolated, as shown in Figure 1, and the governing equations in dimensionless form for a 2-D steady laminar flow of a Newtonian fluid are as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\mathrm{Re}_D}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\mathrm{Re}_D}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) \tag{3}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{\mathrm{Re}_D \cdot \mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

Solution of the above nondimensional, elliptic equations requires that the conditions be specified along the boundaries that enclose the entire flow field. The boundary conditions for the region ABCDEF shown in Figure 1 are as follows.

For section BC, at the inlet, a uniform velocity $u = u_\infty$, $v = 0$ and a uniform temperature are assumed. No-slip boundary conditions are imposed on sections AF and FE. Across the outlet, section ED, zero gradients of all variables in the streamwise direction are imposed. Although this boundary condition is strictly valid only when the flow is fully developed, its use in other flow

Notation

Br	blockage ratio, D/d
C_f	friction coefficient
d	plate spacing
D	plate thickness
G_z	Graetz number, $(D/x)\mathrm{Re}_D\mathrm{Pr}$
$h_{\max}$	maximum heat transfer convection coefficient
H	plate centerline spacing
k	fluid thermal conductivity
Nu	Nusselt number based on hydraulic diameter, $2q''H/(T_W - T_\infty)k$
Nu_D	Nusselt number based on plate thickness, $q''D/(T_W - T_\infty)k$
$\mathrm{Nu}_{D\max}$	Nusselt number based on maximum convection coefficient over the plate, $h_{\max}D/k$
P	dimensionless pressure, $p/(\rho u_\infty^2/2)$
Pr	Prandtl number, ν/α
q''	wall heat flux
Re	Reynolds number bases on hydraulic diameter of parallel thin plates, $2u_\infty H/\nu$
Re_D	Reynolds number based on plate thickness, $u_\infty D/\nu$
T_m	mean bulk temperature
T_w	wall temperature
u	velocity component in x-direction
U	dimensionless velocity in x-direction, u/u_∞
u_∞	free-stream velocity
v	velocity component in y-direction
V	dimensionless velocity in y-direction, v/u_∞
x, y	streamwise and transverse coordinates
X	dimensionless streamwise coordinate, x/D
x_r	reattachment length
Y	dimensionless transverse coordinate, y/D

Greek

α	thermal diffusivity
ν	kinematic viscosity
θ	dimensionless temperature, $(T - T_\infty)/(q''D/k)$

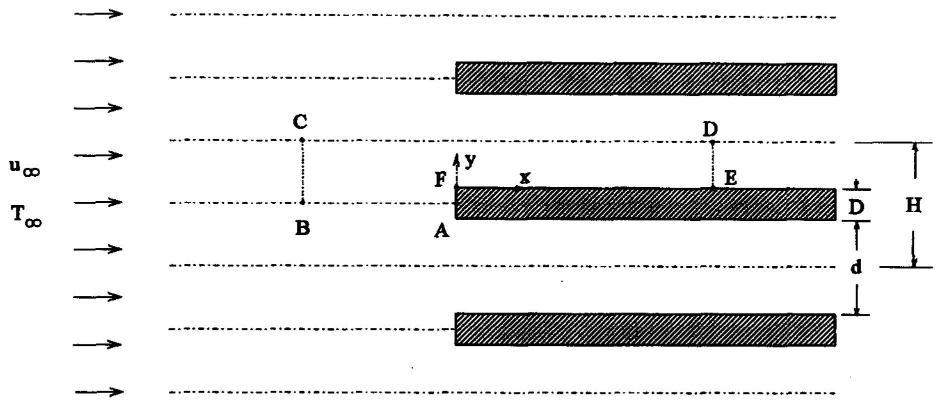


Figure 1 Schematic of the stacks of blunt parallel plates and flow configuration

conditions is also permissible for computational convenience, provided that the outlet boundary is located: (1) in a region where the flow is in the downstream direction; and (2) sufficiently far downstream from the region of interest. Along the centerline CD, $v = 0.0$, $\partial u/\partial y = 0.0$, $\partial p/\partial y = 0.0$, and $\partial\theta/\partial y = 0.0$. On the surface of the plate, a constant and uniform heat flux is imposed.

Solution procedure

The governing Equations 1–4 are discretized using a finite, control-volume procedure. The BSHD is employed for discretizing the convective terms in the momentum and energy equations (Benodaker *et al.* 1985). By considering the local direction of the flow, the BSHD scheme greatly reduces false numerical diffusion that results from nonalignment of the coordinate grid with the flow direction (Raithby 1976; Lai and Gosman 1982). A staggered nonuniform grid is used for the present computations, as illustrated in Figure 2. Solution is obtained by an iterative method together with the pressure correction algorithm PISO (pressure implicit split operation) of Issa (1982). The main advantage of this technique is the high rate of convergence of the numerical calculations for continuity and momentum equations. A modified computational TEACH-II code (Benodaker *et al.* 1983), which incorporates the BSHD/PISO, is used in the present calculations. In this study, the set of difference equations are solved iteratively line-by-line in conjunction with the Thomas algorithm. Computation was started by first solving the continuity and momentum Equations 1–3 to determine the flow field and then the energy equation to find the thermal field in the region of interest.

The computational region extends 5D upstream and 12D downstream from the leading edge in the x-direction and half the plate spacing in the transverse direction. Expanding the

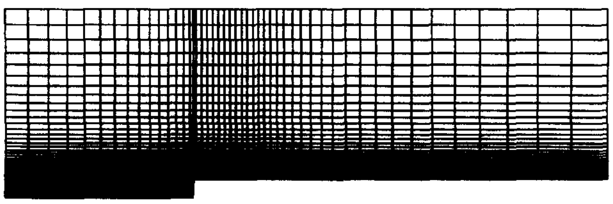


Figure 2 Typical grid configuration of the computational domain

computational domain revealed no significant changes in the flow and thermal field results. The number of grid points in the x- and y-directions were 80 and 40, respectively. Fine and uniform grid spacing are used in the recirculating region and variable grid size on the rest of the domain. Variable grid spacing is used to resolve steep gradients of velocity and temperature near the wall in the y-direction. The configuration of nonuniform grid arrangement is determined from the result of the preliminary studies of laminar flow inside a duct and compared with those obtained by Djalili (1994). It was found that the flow field is almost grid independent when fine grid is used for determining the reattachment length. This is mainly because of the advantages of the BSHD discretization scheme. However, calculations of heat transfer coefficient close to the leading edge are very sensitive to the grid expansion ratio, especially for fluids with high Prandtl numbers. This is mainly because of the thinness of the thermal boundary layer and singularity of convection heat transfer coefficient at the leading edge. By moving away from the leading edge, the convection heat transfer coefficient varies smoothly, and the value of maximum Nusselt number becomes almost independent with small variation of grid spacing. In the present simulation, it was found that the solution is reasonable by taking the grid expansion ratio greater than 0.9 and less than 1.1 in the region of steep gradients. In this study, the range of Reynolds number, Prandtl number, and blockage ratio studied are as follows:

$$100 < \mathrm{Re}_D < 400, 0.7 < \mathrm{Pr} < 10 \quad \text{and} \quad 0.083 < \mathrm{Br} < 0.40$$

The numerical scheme started with an initial condition of uniform velocity and temperature for all grid points in the domain. For some cases, an attempt is made to accelerate the convergence by applying different initial conditions. For example, at higher-Reynolds number, the solution obtained from a lower-Reynolds number was used as an initial condition. Convergence criteria were satisfied when the maximum value of residuals for mass, momentum, and energy were less than 2×10^{-3}.

For validation purposes, present results were compared with the solution of air flow with Pr = 0.7 in the entrance section of a two-dimensional channel. Figure 3a compares the predicted velocity profile and the results obtained by Shah and Bhatti (1987) at several locations along the channel. Predicted Nusselt number in the entrance region of a straight duct is also compared with the solution of Shah and Bhatti in Figure 3b. It is evident from these figures that the numerical prediction for velocity profile and Nusselt number in the entrance region of the 2-D flow

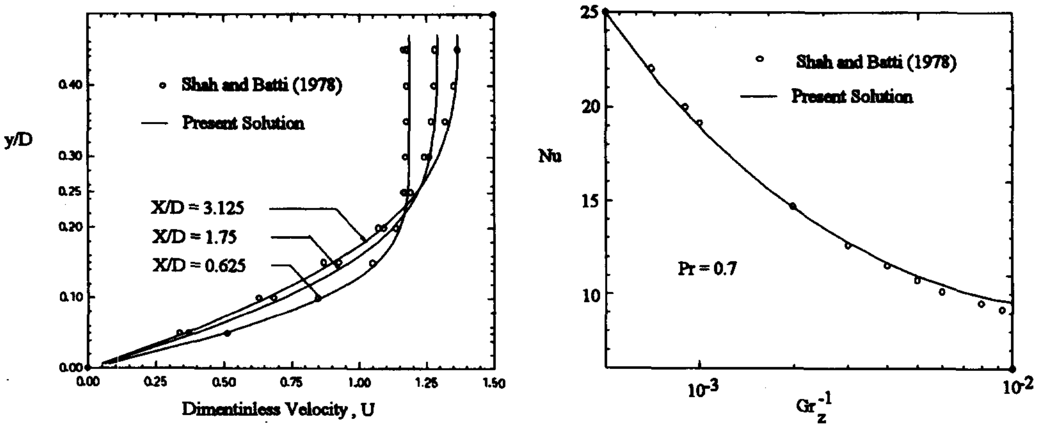


Figure 3 Comparison of (a) velocity profile and (b) Nusselt number, in the entrance region of parallel thin plates

through thin plates are in excellent agreement with the available results in the literature. This comparison reveals that the numerical model can accurately simulate the velocity and temperature distribution in the entrance region of parallel plates.

Results and discussion

Flow patterns

The flow field for the present problem in Figure 1 can be divided into three regions; first the upstream section where the flow approaches uniformly the leading edge of the plates. In this zone, the flow splits into two parts as it approaches the plates, with stagnation point at the forward face of the plate. The second region is the distance between the recirculation and reattachment point of the flow, and the third region is the flow redevelopment after the reattachment point. In the latter region, the boundary layer grows freely along the plate until it is fully developed at about one-half of the plate spacing. There is a slight pressure reduction at the leading edge caused by flow acceleration and adverse pressure gradients in the recirculating zone. Pressure reduction continues as the fluid flows through the plates. The reattachment point is determined by linear interpolation of the computed wall shear stress distribution along the plates. Typical velocity vector field in the entrance region of the plates for blockage ratios of 0.10, 0.20, and 0.30 and $Re_D = 200$ are illustrated in Figure 4. The corresponding Reynolds numbers Re are 4000, 2000, and 1666. In this figure, the plate thickness is assumed constant, while the plate spacing is decreased. The flow reversal in the recirculation region as well as boundary-layer development is clearly visible over each plate. Figure 4, also shows that fluid acceleration caused by blockage is higher near the leading edge, and streamlines move to the center as flow moves farther along the plates. The length of the bubble decreases by increasing the blockage ratio or decreasing the plate spacing. The reattachment length x_r is a function of both the

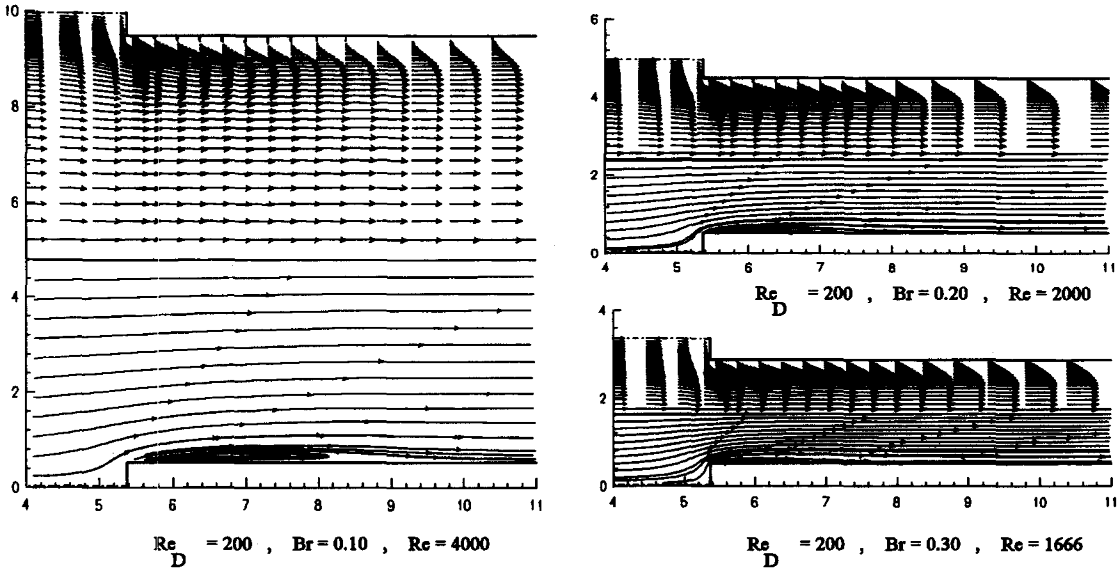


Figure 4 Velocity vectors field and streamlines in the entrance region of blunt plates for different blockage ratios

Reynolds number and blockage ratio. For Br = 0.10 and Re_D = 200, the value of x_r/D is about 2.88. This statement agrees well with those obtained by Djilali (1994). In Figure 5, numerical results as well as the experimental results of Lane the Loehrke (1980) and numerical predictions of Djilali are presented.

Based on the results above, the following correlation is developed for the ratio of recirculation length and its variation with the blockage ratio.

$$\frac{x_r}{D} = 0.0024 \frac{Re_D - 80.0}{Br} \qquad \text{for} \qquad 0.083 < Br < 0.40 \qquad (5)$$

This correlation is also illustrated in Figure 5. The linear variation of recirculation length with Reynolds number and its inverse proportionality with Br are presented in Equation (5). It can be seen that Equation 5 can accurately predict the recirculation length for all practical applications.

Friction

The variation of $C_f = (\mu \partial u/\partial y)_{y=0}/(\rho u_\infty^2/2)$ in the entrance region of the plates for Re = 2000 and different values of Br are shown in Figure 6. In this graph, each blockage ratio belongs to a certain Re_D. Note that Re = 2 Re_D (1 + 1/Br). It can be seen that the blockage ratio has a strong effect on the value of C_f. Friction coefficient has a negative value at the beginning because of the reverse flow close to the wall and zero value at the reattachment point. After the reattachment, C_f increases to a high value. The higher the blockage ratio, the larger the friction coefficient after the reattachment point. This effect is attributable to the reduction of the separation bubble length and the short distance the flow has to develop after the bubble. However, the value of C_f decreases as the flow moves toward the fully developed condition for all blockage ratios. Reducing the blockage ratio, the value of C_f approaches that of thin plates.

Heat transfer

For flow through parallel plates with finite thickness D, the fluid flow and heat transfer depend on Reynolds number Re, Prandtl number Pr, and blockage ratio, Br = D/d. For a combined entry length problem, it is common to plot the results against the inverse of the Graetz number Gz, which is a dimensionless number defined by $1/[(D/x)(Re\ Pr)]$. It should be noted that for flow between thick plates, Nu = 2 Nu_D (1 + 1/Br). With this expression, we may easily convert the Nusselt number based on the plate thickness to the Nusselt number of parallel duct.

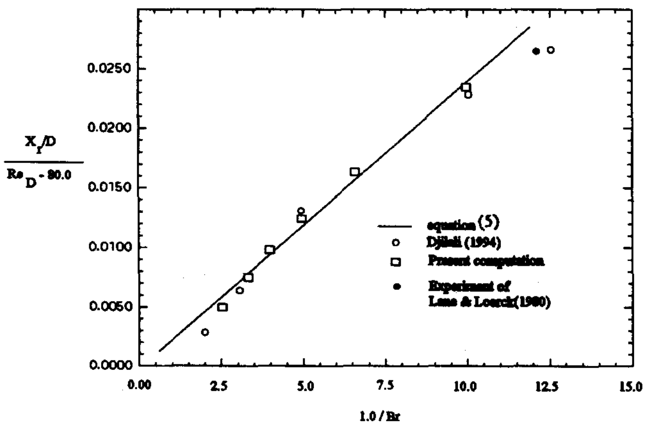


Figure 5 Correlation results for reattachment length

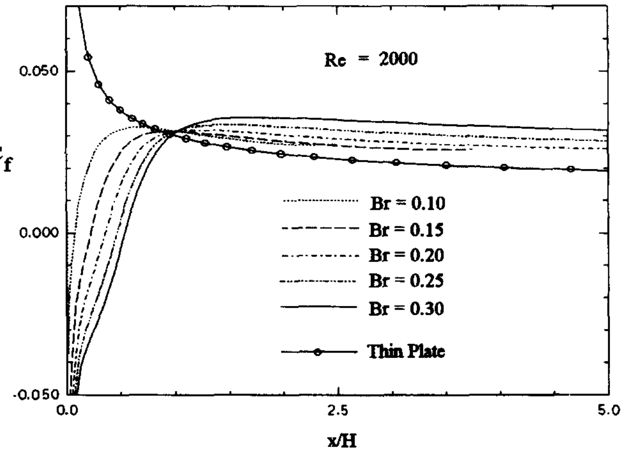


Figure 6 Variation of friction coefficient in the entrance region with blockage ratio compared with thin plates

The value of T_w in the Nusselt number is determined by a third-order extrapolation of temperature at three grid points above the plate. Figure 7 shows the variation of Nu_D for Re = 2000 and different blockage ratios. In this graph, for comparison the Nusselt number for thin plates is also plotted. This graph indicates that as the plate thickness is increased, the corresponding Nusselt number is decreased. However, for all cases, the limiting values of the Nusselt number is the same that should attain a constant value for a fully developed condition.

Figure 8, presents the local Nusselt number variation with respect to Gz^{-1} for a combined entry length problem for flow between plates with a blunt leading edge and for different Re_D, Pr, and Br = 0.20. In all cases, the Nusselt number is high at the leading edge and then decreases for a short distance along the wall, similar to the flow through thin plates, but in these cases, the Nusselt number starts to increase after a short distance and reaches a maximum value at a location close to the reattachment point. After the reattachment, the Nusselt number variation is similar to the entrance region of thin plates. In the bubble, the fluid with the smaller Prandtl number shows a higher heat transfer than those of larger Prandtl numbers (recirculation effects are more significant for small Pr then large Pr). For small

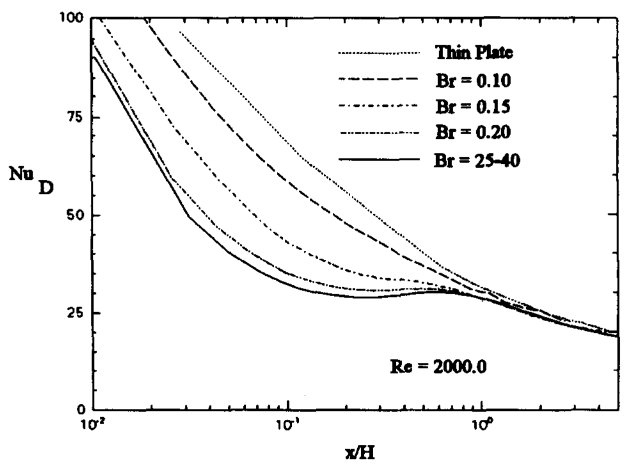


Figure 7 Local Nusselt number in the entrance region for different blockage ratios, compared with thin plates

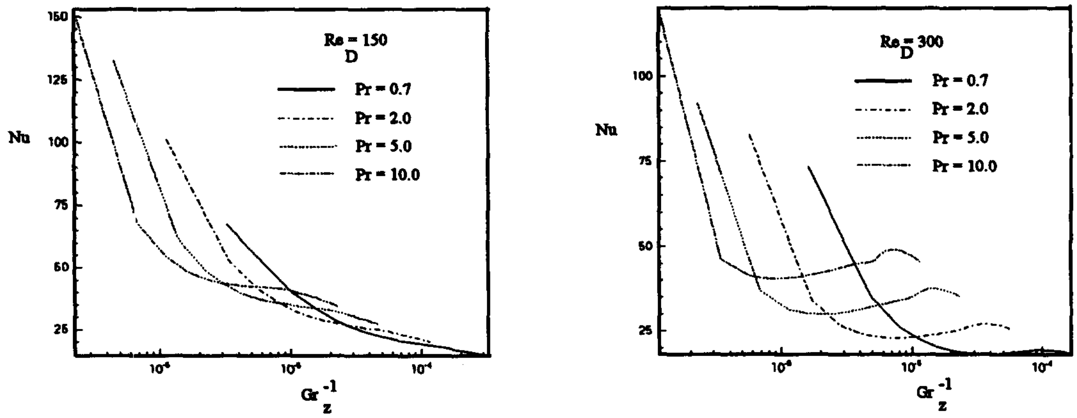


Figure 8 Variation of Nusselt number with Graetz number for different Re_D and Pr

Re_D, the variation of Nu_D, where the Prandtl number is small, approaches the fully developed state at a shorter length. For higher Re_D, the difference in heat transfer coefficient increases. The value of Nusselt number and bubble thermal fluctuation is considerable, especially for high values of Prandtl numbers. These graphs also illustrate that heat transfer is very sensitive to the Pr, because thermal boundary layers are strongly related to the Prandtl number. For all cases, after the reattachment, Nu_D decreases asymptotically for all values of the Prandtl numbers and Reynolds numbers. This value corresponds to the Nusselt number for fully developed laminar flow between parallel plates or ducts with high aspect ratios. This means that for the limiting values, the temperature profile will be fully developed, and the entry length for the thermal field is smaller for fluids, with smaller Prandtl numbers and also higher blockage ratios.

The value of a maximum Nusselt number at the reattachment point is important when we deal with cooling of the plates. For all runs presented, a correlation was developed to determine $Nu_{D\max}$, in terms of Re_D, Pr, and blockage ratio. Figure 9 indicates that there is a linear relation between the ratio of $Nu_{D\max}/(Pr^{1/3}Re_D^{0.216})$, and blockage ratio and the results can be correlated as follows:

$$Nu_{D\max} = Pr^{1/3}Re_D^{0.216}(0.41 + 3.0\,Br) \tag{7}$$

Conclusions

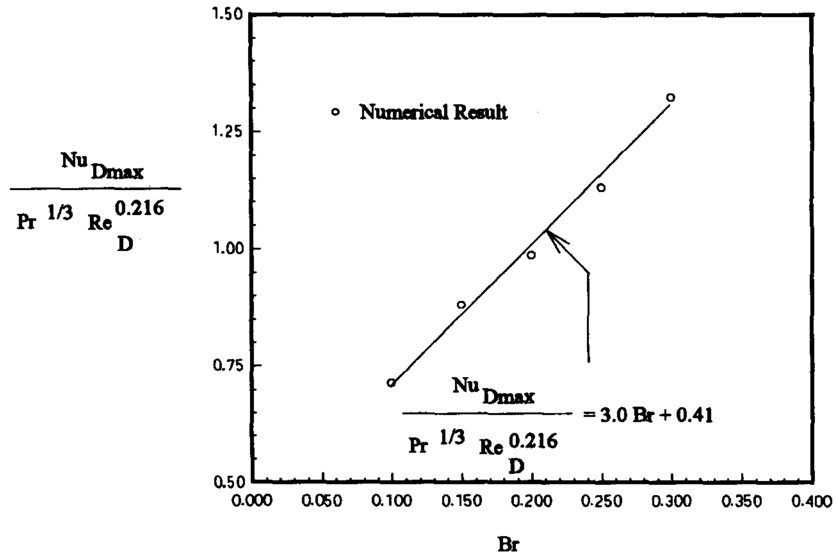
For a numerical study of 2-D flow through an array of thick plates the main conclusions are as follows.

(1) The flow field is strongly related to the Reynolds number Re_D and blockage ratio Br. The ratio of x_r/D can be found from

$$\frac{x_r}{D} = 0.0024\frac{Re_D - 80}{Br} \quad \text{for} \quad 0.083 < Br < 0.40 \tag{5}$$

(2) The friction coefficient C_f along the plates is different from those of thin plates. The higher the blockage ratios, the larger the value of C_f in the entrance zone after about one plate thickness. However, for all cases, as the flow develops,

Figure 9 Correlation of maximum Nusselt number in the entrance region of blunt plates



C_f approaches that of the fully developed condition. For high values of x/D, C_f approaches a constant value, which resembles the fully developed flow condition in the duct.

(3) For plates with a uniform and constant heat flux, convection heat transfer along the entrance zone varies with Re_D, Br, and Pr. The Nusselt number approaches a maximum value close to the reattachment point, and it decreases to the value for fully developed thermal and hydrodynamic condition after the reattachment.

A new correlation of the maximum Nusselt number $\mathrm{Nu}_{D\max}$ in the reattachment region is developed based on present data as follows:

$$\mathrm{Nu}_{D\max} = \mathrm{Pr}^{1/3}\mathrm{Re}_D^{0.216}(0.41 + 3.0\,\mathrm{Br}) \tag{7}$$

Acknowledgment

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